Proc. of 7th Conference of China Numerical Methods on Fluid Mechanics
Changchun, China, 1995, p.50

An Extrapolation that Preserves TVD for Out Flow Problem
(On the Down Flow Boundary)

YU Xin[1], WANG Famin, ZHAO Lie

(Institute of Mechanics, Chinese Academy of Sciences, Beijing 100080, China)

[First paragraph] —- omitted.

When we solve an out flow problem numerically, we cut the infinite solution domain to a finite scale. One method to decide the value on outflow boundary is extrapolation. Zero gradient extrapolation has a good stability with low order accurate; Higher order accurate extrapolations sometimes lose stability. Now we present an extrapolation (on the boundary) that Preserves TVD for out flow problems. It obtains both higher order accurate and a good stability. It can be extended to other schemes.

For a three point TVD scheme, Set U_N^{n+1} on the boundary as

$$U_N^{n+1} = U_{N-1}^{n+1} + \mathrm{sign}(\Delta_{N-\frac{1}{2}}^{n+1} U)\min(|\Delta_{N-\frac{1}{2}}^{n+1} U|, |U_N^n - U_{N-1}^{n+1}|) \tag{1}$$

where $\Delta_{N-\frac{1}{2}}^{n+1} U$ is a first order accurately (linearly) extrapolated value minus U_{N-1}^{n+1}, or a second order accurately extrapolated value minus U_{N-1}^{n+1}, or "min mod"[2] of them, then TVD property is preserved.

Theorem 1. Consider the one dimensional hyperbolic conservation law

$$\frac{\partial U}{\partial t} + \frac{\partial f(U)}{\partial x} = 0 \tag{E}$$

and a $2k+1$ point finite difference scheme of it:

$$U_i^{n+1} = U_i^n - \Delta t \bar{f}(U_{k-1}^n, ..., U_{k+1}^n) \tag{S}$$

[1]E-mail: yu@cerse.psu.edu
Home pages: http://www.imcas.net/yu/

[2]min mod(a, b)=sign(a)min(abs(a),abs(b)) when $ab > 0$, mod(a, b)=0 when $ab \leq 0$

Suppose it is TVD, i. e., for any $\{U_i^n\}$,

$$\sum_{i=-\infty}^{\infty} |U_{i+1}^{n+1} - U_i^{n+1}| \le \sum_{i=-\infty}^{\infty} |U_{i+1}^n - U_i^n| \tag{2}$$

and suppose for any i, $U_{i-1}^n = U_i^n = U_{i+1}^n$ implies $U_i^{n+1} = U_i^n$. Then as along as $U_N^{n+1}, ..., U_{N+k-1}^{n+1}$ satisfy

$$|U_N^{n+1} - U_{N-1}^{n+1}| + ... + |U_{N+k-1}^{n+1} - U_{N+k-2}^{n+1}| \le |U_{N+k-1}^n - U_{N-1}^{n+1}| \tag{3}$$

we have the TVD property preserved, i. e.,

$$\sum_{i=-\infty}^{N+k-2} |U_{i+1}^{n+1} - U_i^{n+1}| \le \sum_{i=-\infty}^{N+k-2} |U_{i+1}^n - U_i^n| \tag{4}$$

Inner point values $U_i^{n+1} (i \le N-1)$ are calculated by the finite difference scheme. Formulae (for k points $U_N^{n+1}, ..., U_{N+k-1}^{n+1}$ on/out of the boundary) satisfying (3) can be given refering to (1).